LECTURES

ON

MATHEMATICAL STUDY,

&c. &c.

THREE LECTURES

ADDRESSED TO

THE STUDENTS OF BELFAST COLLEGE,

ON SOME OF THE ADVANTAGES OF

MATHEMATICAL STUDY.

TO WHICH IS ADDED AN EXAMINATION OF

HUME'S ARGUMENT AGAINST MIRACLES.

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LONDON:

SOUTER AND LAW, 131, FLEET STREET.

GREER, BELFAST; AND MILLIKEN, DUBLIN.

1846.

C. and J. Adlard, Printers, Bartholomew Close, London.

PREFACE.

The following short Addresses were delivered to the Students of Belfast College several years ago. The first and last were given upon public occasions; and the former of these was printed at the time, by request. But neither of them was originally intended to serve any other purpose than the temporary one for which they were expressly drawn up, whatever may have been the Author's hopes as to the abiding influence of any of the sentiments conveyed, upon the minds of the young men for whom, exclusively, they were prepared. The Publisher, however, having repeatedly urged the re-printing of the first Lecture—which, contrary to the Author's expectation, seems still to be frequently inquired for—the request has at length been complied with; and two other Lectures, having the same object, and enforcing the same sentiments, have now been added.

It may be proper to state, in explanation of—not certainly in apology for—the general tenor of these Addresses, that at least nine-tenths of the Students to whom they were delivered, were prosecuting their studies with a view to the Christian Ministry. The Author considered it, therefore, to be peculiarly incumbent on him to keep this fact distinctly before him, in connexion with—and indeed as suggestive of—the course of observations which, in his official capacity, it became his duty to offer. The Notes at the end have been added since: they are intended to illustrate or amplify certain views and statements in the text; and may perhaps furnish a remark or two of interest to the general reader.

BELFAST; *Feb.* 1846.

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ERRATUM.

P. 51, line 1, after the word "error," insert "which."

AN

INTRODUCTORY LECTURE,

&c.

Gentlemen,

It is usual, I believe, in places of public education, to introduce the several seasons of study by means of a Preliminary Lecture.

Upon me has fallen the honour of opening the Mathematical Classes in this Institution, and I now enter upon the duty, not without some share of that confidence which every Englishman feels when about to address an Irish audience; for, great as are my own misgivings as to my claims upon your attention, yet I know enough of Irish character, and have experienced enough of Irish generosity, to persuade me that I shall not, on this occasion—an occasion when I feel that I so much need your indulgence,—solicit that indulgence in vain.

I must apologise to you not only for the manner, but also for the matter, of the observations which I am about to submit to you, for I have thought it advisable to depart in some measure from the course generally adopted upon occasions like the present.

Introductory Lectures are usually devoted to the history of the subject to be introduced. It is not, however, my intention in the present brief discourse, to offer to you anything like an historical account of the rise and progress of so extensive a department of human knowledge as the mathematical sciences embrace. Such an account would necessarily be imperfect in the extreme, and could convey to you little more than a mere chronological statement of the names of those illustrious men to whose accumulated labours, through a period of at least 3000 years, we are indebted for the splendid array of intellectual achievements which constitutes the science of modern Europe—achievements which no one can contemplate in their full extent, without being amazed at the extraordinary powers of the human mind.

Instead, however, of entering into any historical detail of these, I have thought it would be more acceptable to my audience at large, and unquestionably more acceptable to those among you who may be about to enter upon a course of mathematical study, that I should submit to you a few remarks upon some of the advantages connected with that study;—that I should endeavour to point out the unsoundness of the principal objections usually urged against it;—and that I should,

finally, touch a little upon the subject of communicating mathematical instruction.

It was justly remarked, by Locke, that "if you would have a man reason well, let him learn geometry;" and, indeed, one of the peculiar advantages of this study, considered as a part of a general course of education, consists in its remarkable power of disciplining the mind; of calling forth and invigorating the reasoning faculty, and of inducing a habit of close attention and rigid investigation. So that, although a person may care but little about the properties of figures, lines, and angles, which it is the more immediate object of geometry to teach, useful and attractive as this knowledge is, yet, if he wish to sharpen his perceptive powers, if he desire to reason well himself, and to detect the sophistry of others, it behoves him to study geometry. It is, indeed, principally with a view to these advantages that this study is introduced so generally into our college systems of instruction, it being adopted more as a guide to the art of reasoning than as an introduction to the art of measuring.

If we inquire whence arises the superior claim of geometry to our attention as a system of logic, we shall find it to originate in this, that geometry teaches by example what other systems teach merely by precept: these prescribe the rules, while geometry furnishes the most perfect models of their application.

Every one who opens a book on geometry for the first time, and who examines the principles upon which the whole rests, cannot fail to be struck with their extreme simplicity, and can scarcely believe that from truths so simple and obvious, such important consequences could be deduced by any process of mere human reason.

Yet the whole fabric of mathematical science is based upon the axioms of geometry. We may consider these as comprising the terms of stipulation entered into between the student and the teacher; they are the conditions upon which they agree to set out on their voyage of discovery in the pure regions of abstract truth, and to these conditions they engage to adhere throughout their progress. If the preliminaries be objected to, then, of course, no advance can be made; but they are so few in number, and so self-evident in their character, that to refuse our assent to them would indicate a lamentable deficiency of mental apprehension. No one in his senses could possibly refuse to admit that the whole of anything is greater than a part; or, that if equal quantities be equally increased, or equally diminished, the results will still be equal.

These, and such like simple principles, being admitted, the geometer then undertakes to render equally evident the fact that the three angles of a triangle, construct it as you please, modify its shape as you will, shall always

amount exactly to two right angles, neither more nor less; that if you construct a triangle of any magnitude, and under any circumstances, provided only that you secure to it one right angle, he engages to show you that a square, described upon the longest side, shall be exactly equal to the two squares described upon the other sides. In undertaking this, he is willing to occupy the most unfavorable position possible; he requires from you the exercise of no prejudice in his favour; but, on the contrary, would invite you to indulge the most scrupulous frame of mind—to use your utmost penetration to discover a flaw,—nay, to put your whole ingenuity in opposition to him; and yet he will engage to convince you of these, and of all the other truths of geometry, in spite of all your opposition. He will do this, not indeed, without argument, but certainly without the least disputation; for, as soon as you oppose an objection to any of the successive conclusions which he will call upon you to admit as he proceeds in his argument, he will in a moment show you that such objection involves in it an objection also against the very axioms which, at the outset, you agreed to admit; you must therefore withdraw your objection, or else break your original engagement, which is, of course, forbidden. You are therefore *compelled* to admit the inference; and, for a like reason, you must admit all the succeeding inferences, till you are constrained eventually to yield assent to the truth to be established. Such is the irresistible power of demonstration!

Now, every one must feel that there is something exceedingly captivating in this mode of acquiring knowledge; and that there is, moreover, a degree of satisfaction in the knowledge itself, greatly surpassing that attendant upon other attainments. We cannot but feel that what we thus acquire is real truth, subject to no abatement—no modification, depending upon no hypothesis of man—upon the authority of no great name—that these truths will be the same a hundred years hence as they are at present, standing immutable amidst all the changes of systems and all the fluctuations of opinion.

How fortunate is it that the results of mathematical investigation *are* such immutable truths! In what a precarious state would those practical arts be whose very existence is staked upon these results, if they were not thus immutable. Look at navigation, for instance, the science to which the prosperity of this mighty empire is so much indebted; in what state would it be if its mathematical principles were subject to the smallest change or imperfection? How few would be found so venturesome as to depend upon them, if they could not do so with the same implicit confidence that they can depend upon the unerring mechanism of the heavens themselves!

But, even if the mathematical sciences were not of the vast importance they confessedly are in the practical affairs of civilized life, even if they possessed not that power of invigorating the mind which the ablest statesmen, the soundest divines, and the acutest philosophers know by experience they do possess, still they would deserve a large share of attention on account of the beautiful and striking truths they unfold, viewing these merely as matters of intellectual gratification. There is something striking even in the simple truth which I have just adverted to—that the three angles of *every* triangle can never fail to make exactly two right angles, a fact of which every one, who has read the 32d proposition of Euclid's first book, is as well assured as he is of his own existence. To ascend a little beyond first principles: Is there not something striking in the fact that there are certain spaces or surfaces of infinite length, so that, if we were to attempt to ascertain that length by actual measurement, and had even gone on to the distance of the remotest fixed star, we should still be no nearer to the end; and yet that mathematicians know how to determine the exact measurement of these spaces even to the millionth part of a hair's breadth? Is there not something striking as well as interesting in the fact, that a certain curved declivity may upon mathematical principles be formed, possessing the remarkable property that if you suffer a smooth ball to roll down it, the ball will always arrive at the bottom in the same time, from whatever height you let it go; so that, if you observe the time occupied in falling when you let the body go within ten yards of the bottom, you may be perfectly assured that that will be the time in which the body will reach the bottom, if you let it fall from the height of 10,000 miles? This is a truth as well established as that the three angles of a triangle amount to two right angles, the evidence of each truth being equally indisputable, although the former, not lying so near to first principles as the latter, requires that we pass over a greater number of subordinate propositions before we can arrive at it.

I can well conceive how startling the proposition in question must appear to a person unacquainted with the remarkable truths which the higher mathematics reveal; and I can easily imagine how unhesitatingly he would reject a statement so entirely opposed to his preconceived notions of what would take place. What! he would exclaim, do you mean to assert that if I were to let a body fall from the height of this ceiling along the curve you propose, supposing all friction and atmospheric resistance to be removed, that it would arrive at the ground no sooner than if I had let it fall down the same curve from the height of the fixed stars? Is this what you really mean to assert to be as demonstrably true as that any two sides of a triangle are together

greater than the third side? Every mathematician would answer—it is: nay, more; that this very curve would carry a body from any point in it to another lower down in less time than if the body took any other path whatever, although, perhaps, you might suppose that the straight line from the higher to the lower point, as it is the shortest distance between them, would be the line of swiftest descent.

These are but one or two of the very many extraordinary truths revealed to us by mathematical investigation, which, as before observed, would recommend itself to our notice, were it only for the mental gratification consequent upon the discovery of such truths. But it doubly recommends itself to our attention, for the collateral advantages which the mind derives from the reception of these truths. Is it not natural for a person who is guided to such remarkable results by the unerring light of demonstration, sometimes to look back upon the period when, for want of that light, he would have opposed, as absurd or impossible, what he now receives with the fullest conviction? And is it not likely that the question may sometimes arise in his mind—may not other things, things in which I am more deeply interested, notwithstanding their apparent impossibility, be equally true? I admit the propositions of science, because my mind has been led to them by a process of logical deduction. Do I reject the grand propositions of religion because they are not susceptible of being proved, or is my intellect mighty enough to grasp the reasoning, even if it were offered? May not that which is matter of *faith* to mortal minds, be subjects of the clearest *demonstration* to a higher order of intelligences?

No subject is better calculated to excite such reflections as these than the mathematics; and therefore those persons do greatly err, who charge these inquiries with a tendency to create scepticism on the subject of Divine Revelation. By what argument they arrive at such a conclusion I cannot tell. It is true that now and then the anomaly presents itself of an infidel mathematician, but surely such an event can furnish no argument against the study of mathematics, which might not with equal force, at least, be directed against the pursuits of elegant literature; for those eminent cultivators of it, Voltaire, Hume, Volney, and Gibbon, were as eminent unbelievers. If, indeed, this charge against the mathematical sciences be true, how is it that the greatest mathematicians are not always the greatest sceptics? The life of Newton—the prince of mathematicians, was one uniform illustration of the opposite character. He is said to have devoted more time to the Bible than to any other book, and to have been ready upon all suitable occasions to rebuke levity in matters of religion. It is recorded that Dr. Halley, having once ventured in the presence of Newton, to use some irreverend expressions towards the

Bible, was rebuked by him to this effect: "Dr. Halley, if you will converse with me upon the subject of mathematics or astronomy, I will listen to you with pleasure, because these are matters which you well understand, but speak not of the Bible; I have studied it, you have not; and I am well convinced that you know nothing about it."

Newton, indeed, was too well accustomed to reason from analogy to suppose that his Creator would lift the veil from the face of nature, and so largely unfold to him the laws by which He governs the material universe, and yet leave him in the dark respecting all that concerned the moral government of himself; or that he should be gifted with light to perceive the mystic links which connect this dark and distant earth with the glorious source of light and heat, and yet be furnished with no ray to reveal to him the connexion between the eternal purposes of God, and the immortal destiny of his own soul.

As connected with these remarks, it may not be out of place to advert here to the fact, that some persons, but imperfectly versed in scientific subjects, have fancied that they have discovered, in the inspired writings, statements which directly militate against the conclusions of modern astronomy. Thus, they point to the earth's standing still, and justly argue that if such an event had ever occurred, the earth must necessarily have been drawn into the sun, and have been incorporated with its mass, provided the Newtonian law of attraction existed. But surely He who had power to put that law in action has also power to suspend it. There was not, however, any occasion for the exercise of such power; the event might have been effected, and no doubt was effected, by much simpler means, without the slightest interference with existing laws: for it must be recollected that the miracle consisted in arresting the apparent diurnal motion of the sun, or of prolonging his apparent presence above the horizon. Now it is well known that when rays from any luminous body enters a dense medium, they are bent downwards, or so as to proceed in a direction more nearly perpendicular to the surface of the medium after they enter it than they did before; an eye, therefore, situated within this medium, and receiving the bent rays, will refer the luminous body to a place above its real situation; and the greater the density of the medium is, the greater will be the distance between the true and apparent place of the body.

It is this *refraction,* as it is called, which causes the sun, whose rays, upon entering our dense atmosphere, are bent downwards, always to appear higher than he really is; so that we see him, in fact, long after he has sunk below the horizon. To cause the sun, therefore, to appear stationary while he is actually descending, nothing more would be necessary than to gradually increase the density of the atmosphere, which, by bending the rays more and more, would keep the sun's apparent place stationary, and thus prolong the day. A still further increase in the density of the atmosphere would cause the sun to appear to retrograde, and as in the case of the dial of Ahaz, the shadow of an object would go backward. These events unquestionably imply a special exercise of Almighty power, but certainly no suspension of a law of nature.

I shall mention here one more objection, which is sometimes advanced against the study of the higher mathematics; it is on the ground of their practical inutility.

It is not uncommon to hear people say, "An acquaintance with the principles of geometry and mensuration, and with so much mathematical analysis as may be required in the ordinary routine of practical affairs, may be all very necessary for professional men; but what is the use even to them, of pushing these inquiries so far, and of occupying so much time with subjects which, after all, are little else than matters of barren speculation?" Before replying to this objection, I would first inquire which of the numerous applications of mathematical science to practical affairs did not at one time appear to be but matter of barren speculation? The conical pendulum of Huygens was certainly little better than a subject of barren speculation to him: but when, in the succeeding age, it was applied by Watt as a *governor* to the steam-engine, it proved to be one of the most efficient contrivances for equalizing the supply of steam that could possibly have been divised. The remarkable theorem of the Flemish mathematician, Albert Girard, technically known by the name of *the theorem for the Spherical Excess*, was certainly to him, and no doubt to thousands of others, a matter of barren speculation. It was, however, after the lapse of 150 years, at length called from its repose among speculative truths, by General Roy, at the suggestion of the late Professor Dalby, of Sandhurst, and successfully applied by him, during the great trigonometrical survey, to abridge certain laborious computations which occurred in reducing the observations.

And what were the researches of the ancients respecting the conic sections but barren speculations to them and to many succeeding ages? The properties of the ellipse and parabola were as well known in the days of Apollonius as in the time of Kepler and Newton, yet it was not discovered till then that these hitherto unappropriated properties were necessary to the true explication of the celestial motions.

It was by contemplating a certain problem which appeared to be but a barren speculation, and to offer no reward except the gratification of solving it, that Descartes was led to the discovery of his beautiful system of analytical geometry, one of the most useful inventions of modern times, to which the continental

mathematicians are much indebted for their recent rapid progress in mathematics and mechanical philosophy. Our celebrated countryman, Dr. Brook Taylor, to whom we owe the remarkable theorem which bears his name, Taylor's theorem, little suspected the practical importance of that theorem when engaged in its investigation; and, indeed, it was, by British mathematicians generally, regarded, for upwards of half a century, as little more than a barren speculation. But, upon this very theorem, so long neglected in the land of its birth, the French mathematicians have founded a most extensive body of science, as full of valuable applications as of beautiful theories.

These few instances may serve to show that mathematical inquiries, even as regard their practical application, should not be condemned as useless, merely because the state of our practical science is such that they do not find their immediate application. What to one age may appear but barren speculation, may to the next furnish information of the highest practical importance.

But it is the object of these remarks to set forth the advantages of mathematical study, more as a valuable branch of *general* education, than as an essential part of *professional* study. The course to which the present Lecture is introductory, will detail the peculiar objects, exhibit the peculiar powers, and explain the peculiar applications, in the arts both of peace and war, of the several distinct departments of mathematical science. It may be proper, however, even here to notice somewhat more particularly its two principal divisions,—the ancient geometry and the modern analysis.

The geometry of the ancient Greeks has descended to us in a form susceptible of but little improvement: succeeding ages have indeed enlarged its truths and greatly extended their application, but, as regards the elements of the science, the very same course of instruction which was employed in the academies of ancient Greece 2000 years ago, is that which is adopted in our universities of the present day. No book ever enjoyed such universal and continuous celebrity as the collection which forms the Elements of Euclid. It is known and admired throughout the civilized world, and no written language exists which is not enriched by the possession of this masterly work.

Great, however, as are the excellences of Euclid, still it ought not to be regarded as a perfect work. It is not, indeed, to be supposed that the human mind has gone on progressively improving every other department of knowledge, and has stood still with respect to geometry. Mathematicians have been long aware that great as are the merits of this work, still, in common with every human production, it has its defects; and yet, such is the power of prejudice, that, even at the present day, nearly

all the expounders of geometrical truth are awed by a sort of scientific bigotry from all attempt to improve this work; preferring to teach it in all its pristine excellence, and with all its pristine defects. For, notwithstanding the emendations of Dr. Simson, the great restorer of Euclid, there are still defects, both as regards the arrangement of the propositions and the manner of demonstrating several of them, which are rendered unnecessarily long and tedious, from the restriction which Euclid had imposed on himself of allowing no operation to be considered as possible till he had shown how to perform it—a restriction which, as Professor Playfair justly observed, is altogether unnecessary. Euclid's fifth book, treating on the important subject of Proportion, is universally allowed, notwithstanding its peculiar beauty as a masterpiece of profound reasoning, to be too difficult and subtil for the generality of students to comprehend; it is accordingly omitted in most of our public institutions; indeed, Leslie goes so far as to say that it cannot be taught; and yet every system of geometry which does not teach proportion must be essentially defective; so that it behoves every mathematical teacher, who feels compelled to reject Euclid's system of proportion, to supply some other equally logical and more generally intelligible.

These defects, however, are, after all, but trifling compared with the general excellences of the work. The rigid accuracy of Euclid's reasoning throughout his most intricate demonstrations is truly remarkable. In this same fifth book, for instance, a book requiring the utmost caution, he has never once fallen into any of the logical errors which almost every one of his modern improvers have committed who have attempted to demonstrate his very general propositions. In abridging his arguments, they have unconsciously abridged also the generality of his conclusions. Leslie's fifth book is an instance of this; so also is that of Legendre: and in the very last modernised Euclid which has been published in the English language, the author has fallen into a logical error—quite fatal to his conclusions, in the only demonstration in the fifth book where he has departed from the text of Simson's Euclid. In attempting to shorten Euclid's, or rather Simson's demonstration, he makes use of a proposition which holds only in particular cases of that in question, and thus destroys the generality of his conclusion. Those who study geometry without efficient assistance should carefully guard against being misled by this very common error, viz., that of attributing to the whole chain more strength than belongs to the very weakest of its links. I would here, too, caution the student against another error, which loose reasoners frequently commit, and which geometers of note have sometimes fallen into: it is that of confounding the direct proposition with its converse.

Leslie fell into a notable mistake of this kind, in his theory of parallel lines; and it is remarkable, that the able author of the article "Geometry," in the Encyclopædia Metropolitana, has committed a like error when treating on the same subject. We find it demonstrated, in that article, that *the three angles of a triangle, taken together, are equal to two right-angles* and from this proposition the author inadvertently deduces, as a corollary, this inference: "It follows from this, that if two lines are cut by a third, so as to make the two interior angles less than two right-angles, these lines produced, will meet and form a triangle:"—an inference which is quite unwarrantable, and which is, in fact, the converse of the true inference, viz. that *if two lines which meet when produced be cut by a third, the two interior angles will be less than two right-angles.* It is unquestionably most illogical to argue that because the angles which a line forms with two meeting lines have a certain property, therefore, conversely, whenever that property obtains the lines must be meeting lines; as well might we say, that because when it rains the ground is wet, therefore, when the ground is wet, it rains.

In pointing to these oversights, it will not, of course, be suspected by any one here, that I am actuated by the least desire to deduct from the well-established reputation of such men as Legendre, Leslie, and Barlow; such an attempt from me would be as futile as it would be presumptuous. I have preferred to take my examples from the writings of men of the first-rate scientific character, in order the more powerfully to impress upon the student's mind the necessity of using the most scrupulous caution, throughout all the steps of a demonstration, lest he be betrayed, through inattention, into the admission of similar logical absurdities. British geometers do not often commit these inaccuracies; they are, in general, much more scrupulous in their logic than their continental neighbours. Indeed, in no country has geometry been so ardently cultivated,—so extensively applied,—and its powers so severely tasked,—as in the British empire. The wonders it had effected, under the guidance of Newton's genius, seem to have drawn English mathematicians into the unfortunate mistake of attributing to the instrument much that was really due to the extraordinary skill of the operator; and thus the efficiency of geometry, as a means of mathematical research, came to be greatly overrated, and too exclusively employed; for, while the English were thus labouring upon an almost exhausted soil, which but ill repaid the time and talent expended upon it, the continental mathematicians, by cultivating the fertile field of analysis, were reaping the reward of a rich and abundant harvest: while the researches of the French and German mathematicians were continually enriching science by the most important discoveries;—while Clairaut and Euler,

Lagrange and Laplace, seizing as it were the veil, as it dropped from the hand of Newton, and, aided by the powers of analysis, were disclosing to us more and more of the mechanism of the universe; the British mathematicians were, with but one or two exceptions, still fruitlessly occupied with the ancient geometry, unwilling to believe that what had effected so much was not adequate to accomplish still more.

During the present century, however, analysis has been ardently and successfully cultivated in this kingdom, and a complete revolution has at length taken place in the mathematical studies of our collegiate schools. Geometry is now confined to its legitimate purposes, and is applied only to those inquiries which fall fairly within the compass of its powers, while all the more intricate researches of mechanics and physical astronomy are submitted to the more effective dominion of the modern analysis.

The powers of this analysis, as an instrument of mathematical investigation, seem, indeed, to be almost unbounded; even things which are totally inconceivable in themselves, may properly become the subjects of its reasonings; for, we here reason not with the things themselves, as in geometry, but with certain symbolical representations of them; so that the mind is, during the process, relieved, and, indeed, entirely withdrawn from all perplexities about the things represented. Combine our data as we will, provided only we commit no logical error in so doing, the results we shall obtain will furnish so many facts respecting the things represented, be these what they may—conceivable or inconceivable—possible or impossible.

To persons unacquainted with mathematical investigations, the difficulties of certain inquiries appear to be altogether of a different character from those which really perplex the mathematician. Talk to such a person about the striking results of physical astronomy, or that department of applied mathematics which investigates the motions of the heavenly bodies, the laws which govern these motions, and the mutual perturbations consequent upon them, his notion of the difficulties of such inquiries will be associated with his ideas of the vast magnitude of the bodies reasoned about—their immense velocities—their unapproachableness; whereas, these circumstances affect not the mathematical difficulties of the investigation in the smallest degree; these would remain the same, if the bodies were no larger than peas, moving ever so slowly within the narrowest limits.

This arises from the very comprehensive nature of algebraical reasoning, which confines itself not merely to the particular case to which we wish to apply it, but embraces the whole species of which that in question is but an individual example. Thus, the mathematical results of any investigation into the planetary motions furnish us with facts not only respecting the actual system of the world, but also respecting a whole class of systems, of which ours may be regarded but as one solitary case.

Even when, by giving the suitable fixed values to the symbols, which, in our general result, are arbitrary, so as to restrict that result to the particular case in question, still the result thus restricted will not unfrequently convey to us much information respecting the matter of inquiry, over and above that which it was our more immediate object to obtain. Thus, suppose that, setting out with the Newtonian law of gravitation, we wished to determine the curve which any of the planetary bodies describes about the sun, the result of the analytical investigation would show us that these curves, which we already know to be comprised within finite limits, must all be ellipses, of which the sun or seat of force occupies the common focus.

But the result would show us more: it would show us that the same law of force which governs the planetary motions, causing them to describe ellipses, would have been equally competent to have caused them to describe either hyperbolas or parabolas, curves which do not return into themselves, like ellipses, but whose branches extend onwards to infinity. Hyperbolas are, indeed, the curves in which certain comets are strongly suspected to be moving, although these bodies have in general, like the planets, elliptical orbits.*

Now, if we wish to know how it happens that the planets of our system all describe ellipses instead of parabolas or hyperbolas, we shall readily satisfy our curiosity by examining our algebraical formula, which will show to us that the precise curve, depends not upon the caprice of chance, but entirely upon the velocity and direction with which the body was, at the creation, dismissed from the hand of its Almighty Creator. If this velocity had been equal to a certain assignable rate, depending, indeed, upon the original distance of the body from the sun, the path would have been a parabola: a velocity greater than this, in the same direction, would have caused the body to describe an hyperbola, while a

* Sir John Herschel states the hyperbolic motion of a very few comets to be an ascertained fact: he says, "A very few comets have been ascertained to move in hyperbolas." (*Treat. on Ast.* p. 307.) Without venturing to question a statement from so high an authority, I must confess that I know not to what modern astronomer we owe the settlement of this point. Laplace observes, "The hypothesis of the parabolic motion of comets is not perfectly correct; the probability of it is even extremely small, considering the infinite number of cases producing an elliptic or hyperbolic motion, in comparison with those producing a parabolic. Besides, a comet, moving either in a parabola or hyperbola, would be visible but once; hence we may suppose, with great probability, that the comets describing these curves, if there be any, have disappeared a long time since." (*Mécanique Céleste*, book ii.)

less velocity would send it forth in an elliptic orbit. Hence, for aught we know to the contrary, there may exist planets whose motions are continually governed by the sun's influence, but which may have been launched into space at their creation with a velocity and direction capable of prescribing to them parabolic or hyperbolic orbits, and which, therefore, are continually progressing, in the remotest regions of immensity, without ever returning; and yet so completely under the control of the sun, that the slightest change in his attractive influence would divert them from their wonted paths.

But it is in combating the difficulties of the planetary perturbations that analysis can boast of its richest triumphs. It has made known to us the extraordinary fact, that the inequalities in the planetary motions, arising from their disturbing influences upon each other, and which, by accumulating age after age, seemed destined to accomplish the destruction of the universe at some remote period, are all confined within certain limits, beyond which they cannot reach, and within which the stability of the whole system is secure!

The planetary orbits thus eternally oscillate to and fro, ranging to a certain limited extent on each side of a mean position, from which the deviation can never become sufficiently great to endanger the security of the system. Each oscillation, though comparatively small, occupies a very long period of time: so that, as a modern

French astronomer has beautifully said—in reference to these oscillating orbits—that they may be regarded as "the vast pendulums of eternity, that beat ages as ours beat seconds." *

In a discourse, of which the object is to recommend the study of the mathematics, a question often asked naturally presents itself to our consideration. Does this study require any peculiar predisposition of mind? Must there not exist a certain intellectual appetite for it—an innate mathematical bias?

I cannot, I fear, give a direct and satisfactory answer to this question. Certain it is, however, that the successful communication of any branch of knowledge presupposes two requisites—a power to communicate on the part of the teacher, and an aptitude to receive on the part of the learner.

I have already taken occasion to remark that when once you get a person to admit the axioms of Geometry, you secure his assent to every step in the subsequent demonstrations; but, of course, this is on the supposition that the language employed is intelligible to him—that he is capable of understanding a logical process; for, if this be not the case, you can neither expect from him his assent, nor yet any competent reasons for withholding it. That such a peculiar inaptitude of mind may exist, we have a notable instance in the case of the

* Pontécoulant.

celebrated Ferguson, who, although he so much distinguished himself as a writer on almost every branch of mechanical philosophy, yet, according to his own confession, was never able to understand a single demonstration of Euclid. His method of satisfying himself of the truth of Euclid's propositions was by very accurately constructing the figures on a large scale upon pasteboard: the parts to be compared were then carefully cut out, and accurately weighed, and thus their relation to each other ascertained. If merely angles or lines were concerned, he proceeded by actual measurement. In this way he verified the truths of Geometry, having, it seems, no notion of any more rigorous process. Now, whether this remarkable defect in Ferguson arose from any real infirmity in his powers of perception, or from the sluggishness and unwillingness with which his mind pursued truth by any road except that which long habit had accustomed him to, cannot be confidently determined; but, from the clearness with which he could conceive and arrange in his mind the details of complicated machinery, I should think the latter to be the true cause of his inaptness in this peculiar exercise of his powers.

Now, in the same way that Ferguson learnt Geometry do a very great number of the youth of the present day learn arithmetic. Ferguson was satisfied with the truth of the forty-seventh of Euclid's first book, because, by constructing the figure, and weighing the proposed parts, he found the assertion of Euclid verified. The student in arithmetic, in like manner, too often places his confidence in any proposed rule of calculation, because, by applying it to a particular example or two, he finds his result to be always that which is predicted. But if any one had come to Ferguson, and had said to him, "It is very true that, in by far the greater number of right-angled triangles, the square on the longest side is equal to the sum of the squares on the other two, yet, there do exist a very few right-angled triangles which have not this property, and the reason that you are unacquainted with the fact, arises from the circumstance that you have not happened to fall upon one of these in your experiments." If any one had asserted this, Ferguson could not have contradicted him with confidence, seeing that he had not *weighed* every possible right-angled triangle.

Those who receive the doctrines of arithmetic upon the same kind of partial experimental proof are in the same predicament; their knowledge is always uncertain and insecure; and, in acquiring this imperfect knowledge, they acquire at the same time a habit likely, as in the case of Ferguson, to indispose the mind to the pursuit of truth by the more slow but more certain path of logical deduction. When properly taught, the study of arithmetic forms an excellent preparative to that of Algebra and Geometry; that is, when its theoretical principles are conveyed as well as their practical appli-

cation; when the rule is shown to harmonize with the reason, and when the judgment and reflection are called into exercise as well as the memory.

The system of learning arithmetic by rote not only opposes the development of the reasoning powers, and discourages the exercise of thought, but it also imposes a weary drudgery upon those who have the misfortune to be subject to so fruitless a system. I have some painful recollections of having been myself crammed with figures upon this plan; and although, when at school, I got the credit of being a tolerably expert calculator, yet I well recollect that I could never work out a single question promiscuously chosen till I had asked the proposer "*what rule is it in?*" Even to read the question before I had obtained this information seemed to me a needless task, and to read it afterwards seemed almost as needless, so intimately was the rule associated with the mechanical operation.

Such a system as this cannot be too strongly deprecated. It shall not meet with the slightest countenance in the mathematical department of this Institution. No such libel will here be passed upon the mathematics, as to call any subject a branch of it which conveys results without reasons. I am the more urged to this from reflecting that, although gentlemen, whose talents and ardour will ultimately procure for them a conspicuous rank among mathematicians, may still continue to resort here, yet there are not a few whose principal object will be attained if they can but acquire the habits of mind which mathematical studies are fitted to produce; who would even be content to forego the truths of Geometry, if they can imbibe the spirit of the reasonings by which they are established; who in fact expect, like invalid travellers, to benefit more from the journey than from the resting-place to which it conducts them.

That our system may harmonize with the present improved state of science, much that is here taught, after the elements of Geometry, will be taught analytically, and in a manner which will, I trust, show that the processes of analysis, even in its highest departments, are, when properly conducted, as rigorous and conclusive as the steps of a geometrical investigation, although the contrary has often been asserted. It will be a prevailing principle with me to expound nothing from the Mathematical Chair which is not susceptible of rigid demonstration, or which cannot be rendered intelligible to ordinary minds. It may be all very well for the accomplished mathematician to occupy his leisure in framing mathematical enigmas, and in trying how high above the level of his own understanding the transcendental analysis will carry him; but *we* shall be content with a more limited range, and shall always consider it to be prudent to check our flight when we are beginning to lose sight of the principles of common sense.

It will be, then, my anxious endeavour to explain clearly, and to illustrate copiously, the various formulas with which mathematical analysis abounds; so that the student may be in no danger, while contemplating the symbols, of losing sight of the things signified; a danger to which the solitary student of modern mathematics is unfortunately too much exposed. It was the characteristic excellence of our older mathematical books, that they abounded in practical illustration of the theories taught. Our modern authors are departing more and more from this admirable system; and it is no unusual thing now, in looking through a mathematical book, even upon a practical subject, to meet with nothing but a bewildering array of symbols, without a single illustration of their meaning, or a single example of their practical application. This is more especially the case with works on the higher Calculus, and on Analytical Mechanics, which are little else than abstract algebraical exercises, furnishing no real information to the reader, nor even any evidence that the writers themselves understand their own formulas.

"It must be confessed," says that profound mathematician and philosopher, the late Dr. Thomas Young, "that in this country the cultivation of the higher branches of mathematics, and the invention of new methods of calculation, cannot be too much recommended to the generality of those who apply themselves to Natural Philosophy; but it is equally true, on the other hand, that the first mathematicians on the continent have exerted great ingenuity in involving the plainest truths of mechanics in the intricacies of algebraical formulas; and, in some instances, have lost sight of the real state of an investigation, by attending only to the symbols which they have employed in expressing its steps."

It is to be hoped that such a charge as this will never attach to us; but that we shall be even more solicitous about the accuracy and soundness of the information conveyed, than concerning the mere extent of it. It must not, however, be inferred from these remarks, that the Course upon which we are now about to enter will not be an extensive one, or, that I shall be able so to model it as to render the student's progress free from all difficulty and perplexity. Such a "royal road" must not be expected.

In certain applications of the higher mathematics, even when simplified to the utmost degree, difficulties will doubtless occur; to overcome which will require the exercise of much patient thought, on the part of the student, as well as the skilful application of much anterior knowledge. He, indeed, who aims at distinction in any department, either of literature or science, must lay his account with much personal exertion and much private study; and he who expects to attain an intimate

and extensive acquaintance with mathematical science, simply by attending the lectures here, will, I fear, find himself grievously disappointed. I must honestly tell him, that I shall consider my duty performed if I can but place the subject before him in an intelligible and attractive form,—if I can impart to him a taste for the pursuit. I, indeed, can do little more than place him fairly in the path, direct his first steps, and remove the more formidable obstacles out of his way. I cannot run the race for him, although I may be able, occasionally, to smooth the asperities of the road. His success must be the reward of his own exertions, while the very utmost that I can expect from mine, will be the humble but encouraging praise of having been a useful, a faithful guide.

LECTURE II.

PRELIMINARY TO THE MATHEMATICAL COURSE.

GENTLEMEN,

Before we formally enter upon the actual business of the Mathematical Course, I think it advisable to direct your attention to a few introductory remarks, calculated, it is presumed, to convey to you some distinct notion of the chief objects and advantages which that course contemplates.

Upon your being invited to engage in a new pursuit, it is, indeed, natural that you should look for a preliminary of this kind; in order that you may be enabled to form something like an estimate of the profit which may accrue to you from the time and application to be expended upon it.

An unexplored field of inquiry is recommended to your exertions; and it is both reasonable and commendable that you should pause upon its margin, and inquire whether the harvest which it promises is of sufficient value to repay the labour of its cultivation: whether the probable returns can justify the outlay; or, whatever be the intrinsic value of these, whether with your professional views, and ulterior purposes, they can be of any importance to *you*.

Most of you, no doubt, have indulged in hopes and expectations as to your future condition and advancement in life. Many of you, perhaps, have even anticipated the events which are to minister to and promote that advancement; and have sketched out for yourselves some definite course of action, in which you expect hereafter to engage. You have mapped down, as it were, upon the great chart of human affairs, the route you hope to pursue, and the position you expect eventually to occupy. You have thus, perhaps, identified your own happiness and usefulness with some important object, to facilitate the attainment of which your efforts now are principally to be devoted. Anxious, therefore, as you naturally must be to employ the time now at your disposal, so as to render your future exertions the most efficient in the sphere you have chosen for their display, it cannot be matter of indifference to you to what objects that time is devoted. It is, on the contrary, a subject which demands from you the most careful and deliberate consideration; in order that, in the selection of your pursuits, you may give the preference to those which most directly tend to promote the accomplishment of your ultimate desires.

I need not, gentlemen, attempt to enumerate to you the various advantages of Education, in *all* its departments, to *every* person, whatever be his profession, or whatever his position in society. Every one will admit that there is no situation in life in which it can be predicated of any branch of knowledge, that it shall be useless—even in a practical point of view; while, as a mental acquisition, there can be no doubt that every addition to our stock directly contributes to elevate us more and more in the scale of nature as moral and intellectual beings. On the present occasion, however, our attention is to be confined to a brief view of but one particular department of human knowledge—the science of Mathematics—a department which, while it exercises an immense influence over nearly all the regions of philosophical inquiry, offers at the same time advantages to the student almost peculiar to itself, or at least, advantages which no other branch of education possesses in so eminent a degree.

Education, you will observe, considered in its enlarged sense, has a twofold purpose: its object is not limited to merely storing the mind with useful truths, but it also aims at developing and invigorating the very powers, by the exertion of which all knowledge is to be acquired—thus lessening the toil and irksomeness of the acquisition. It is the high office of Education to strengthen and improve the instrument with which she works—the intellect itself:—to expand its powers—to enlarge its grasp—to sharpen its perceptions. These especial advantages of education it is of the first importance to secure, whatever particular departments of it may be more immediately connected with any particular professional views. It behoves us early to submit our minds to this invigorating discipline, that we may be prepared and strengthened to contend with the difficulties with which every path to knowledge is more or less beset; and especially that we may be empowered to penetrate the dark recesses where error lurks, whose baneful ramifications have sometimes been suffered to spread unobserved, till they have marred and deformed, and debased the loftiest features of Truth herself.

Now it is chiefly to secure to you these intellectual advantages that the mathematical sciences are here recommended to your attention; recommended not so much, as Mr. Locke says, to make you mathematicians, as to make you rational creatures. For, great as are the practical benefits which these sciences confer upon mankind in the most important concerns of civilized life, yet it is to the admirable mental discipline which they furnish that they owe their high rank in the scale of education. There is, perhaps, not a single department of human inquiry so well calculated to develop the reasoning powers, and to impart those habits of

cautious and continuous investigation so necessary to ensure success in every intellectual undertaking of difficulty or importance. Nor is it the intellect alone which these pursuits tend to invigorate and exalt; they exercise an important moral influence; inasmuch as they are peculiarly calculated to imbue the mind with an ardent and uncompromising love of TRUTH, the attainment of which, in all its varied and dignified forms, ought surely to be the paramount object of erring humanity—the aim and end of our moral existence.

Such, gentlemen, are some of the advantages which you are entitled to expect from the Course of study upon which you are now about to enter. That any disquisition concerning geometrical figure and magnitude can bring about such desirable results may seem to some of you a little mysterious; but the mystery will explain itself as you proceed. You will find your judgment rendered more discriminating—your reasoning more accurate—and your capacity for close attention and consecutive thinking more enlarged. You will learn to distinguish argument from sophistry, evidence from assumption, and, above all, you will learn the important lesson, that what may be really incomprehensible may nevertheless be demonstrably true.

Although mathematical science is preeminently the science of truth—rejecting every species of evidence in establishing its conclusions that comes short of absolute certainty—yet it abounds in statements at once incontrovertible and incomprehensible; and in trains of reasoning which, though conducted by the aid of quantities in themselves inconceivable, and implying operations in themselves impossible, nevertheless terminate in results which we know to be true.

What a salutary check, then, must such inquiries as these necessarily oppose to general scepticism! I mean to that presumption in which ignorance arrays itself when it refuses to believe merely because it cannot comprehend. The mathematician reposes with the most entire confidence upon facts which he knows to be incomprehensible, though proved to be true:—with what consistency, therefore, in more exalted and more important fields of inquiry, can he refuse his assent to any statement, merely because it is above his comprehension? To a mathematician, the claim which any conclusion has upon his assent depends not in the least upon the character of that conclusion—neither upon the mysteries it involves, nor upon the doctrines it unfolds—but solely upon the evidence by which it is supported. The mind, therefore, which has been accustomed to mathematical discipline would never account it necessary, before receiving any conclusion, to deliberate upon its tendency, or to speculate upon its consequences, whether immediate or remote. Into a mind so trained there would never enter the thought of pitting against

such conclusion any notion, or prejudice, or theory that might preoccupy the imagination: all that would be looked to, as determining the admission or rejection of the doctrine propounded, would be the evidence on which it rests; this, indeed, cannot be too rigidly examined, nor too severely scrutinized, in every case of importance; but when the evidence thus examined is found satisfactory, the conclusion *must* be received, however unpalatable, or incredible, or mysterious it may appear.

Even among the many results of mathematical investigation which are perfectly comprehensible, some are so extraordinary, that they would be quite incredible were they not established upon the immoveable basis of rigid demonstration; and they are accordingly looked upon as incredible by many who cannot examine the evidence by which they are confirmed. To some of these I adverted on a former occasion,* and endeavoured to point out the salutary effect which our conviction of the truth of such extraordinary facts was calculated to have upon the mind, preparatory to engaging in loftier and more sacred pursuits. For, be it remembered that the extraordinary character of these facts remains as unabated after, as before their validity has been proved to us; so that, instead of perplexing ourselves with vain attempts to unravel what may be

* See the preceding Lecture.

confessedly mysterious, we acquire the more philosophic habit of looking solely to the evidence, of cheerfully sub mitting our minds to whatever it may lead, and willingly prostrating every prejudice before the majesty of Truth.

In speaking of evidence, we must not, however, forget to draw a distinction between that which is mathematical and that called historical. We have here to do only with the former, which is of a character altogether unquestionable, and is nothing short of absolute and irresistible demonstration. The truths, too, to which it testifies are in themselves unalterable and eternal:—they necessarily exist, and could not be otherwise than what they are.* Historical evidence, on the other hand, depends upon the faithfulness of tradition—the credibility of human testimony in reference to facts that might have been otherwise. There is generally a third kind of evidence mentioned, and called experimental evidence, or that derived from actual observation. Now, I must confess that I do not very clearly see this third distinction. In fact, upon a close examination of fundamental principles, it will, I think, be found that no department of knowledge is so very abstract and ideal as to be totally independent of experience and observation, or of all information derived through the senses. Even in geometry, although the deductions of pure reason furnish all the materials for

* See Note (A) at the end.

the vast superstructure which it presents, yet we cannot deny that the basis of that superstructure is observation, although certainly of the very simplest and most accessible kind. The axioms of Geometry are, in fact, *indemonstrable* truths; and what truths, it may be asked —truths known and felt to be such—can possibly be acquired independently alike of a reasoning process, by which they become inferences from other truths, and of observation—by which they become the immediate objects of the senses? I need scarcely say that I put this question exclusively in reference to what mere human effort can accomplish, independently of inspiration or of divine revelation. The axiomatic principles adverted to carry to the mind, with their very announcement, an immediate conviction of their truth; and they do so because everybody's experience, however limited, is sufficient for their verification; and it is only to such universal experience that we could refer, if in any instance assent to these truths should be refused:—our final resort would necessarily be an appeal to experiment.

It appears, therefore, that even in pure science, as well as in Natural Philosophy, observation forms the substratum of all our knowledge; and forms, as it were, the first link which supports the entire chain of subsequent deductions. We may thus regard the immense mass of mathematical truth, which the intellectual labour of ages has accumulated, as nothing more than a vast group of splendid corollaries, logically deduced as necessary and unavoidable consequences of the very simplest facts that present themselves to the notice of man.

Historical evidence, too, when it really deserves the name of evidence, is, strictly speaking, the evidence of experience; less irresistible, no doubt, than that by which mathematical truth is established, because this appeals ultimately to our own immediate observation, whereas the former narrates to us the experience of others. It is true, the narrative may be open to suspicion:—we may doubt the veracity of the narrator; but this very doubt is founded on experience—the experience, namely, of the occasional falsehood of human declarations; and, in fact, upon this kind of experience has been constructed a valuable and perfectly sound and practicable theory in reference to the credibility of testimony; and from which theory we find that an event however unlikely to have occurred, if testified to by a few independent witnesses of ordinary veracity, approaches to absolute certainty, with a degree of closeness that could never have been suspected.*

Thus all our real knowledge, whether we derive it from our own individual observation—from the testimony of others—or from the deductions of science—is, in fact, derived from experience. How then can any one entitled to be called a man of science feel himself

* See Note (B) at the end.

authorized in summarily rejecting a statement as untrue, because it rests its claim to truth only upon historical evidence, or traditional testimony, or the recorded experience of others? Such a rejection would be a violation of the soundest principles of the soundest philosophy.

I have been induced to make these remarks for the purpose of meeting a prejudice sometimes entertained by well-meaning persons against scientific pursuits in general, from their alleged tendency to engender habits of mind unfavorable to the reception of a Divine Revelation. The prejudice is a remarkable one certainly, and the notion not a little singular, that the mind which has been trained by the most rigorous discipline to the pursuit of Truth, should, on that account, be rendered unfit for the reception of *Divine Truth*. If these observations have been understood, I think you must perceive that, with a mind so trained, every subject must stand a far better chance of fair play, than with a mind which, for want of such training, is the victim of prejudice; and which rejects every thing that it cannot reduce to the contracted span of its own comprehension. It is true, that the cultivation of science makes us slow to believe what is offered without evidence of its truth, but then it makes us equally slow to reject what there is any evidence to support.

Notwithstanding, however, the general truth of these statements, I do not wish to conceal the fact, that some men of the most splendid mathematical and philosophical attainments have been infidels. The celebrated Laplace, for instance, who was beyond all question the greatest mathematician in Europe, was nevertheless an unbeliever in Christianity. But it would be wretched reasoning to argue, that because great scientific acquirements were found connected with religious unbelief, therefore one was the cause of the other. I would rather say that if Laplace, amidst the illumination of science by which he was surrounded—and which revealed to him a more commanding view of the magnificent machinery of the universe than had ever before been disclosed to the enraptured gaze of man—could, notwithstanding, be an infidel, his infidelity would have taken a far deeper dye had he been left in the darkness of ignorance. But we have no proof that Laplace ever examined the evidences for the truth of Christianity, or tested it by the same rigid maxims that all the articles of his scientific creed were submitted to. Had he brought to the examination of *this* subject but half the honest zeal, and but a small portion of the continuous exertion, by which he sought the evidences for the truth of gravitation among the satellites of Jupiter, the splendour of his scientific fame would not have presented so painful a contrast to the gloom of his religious feelings. His error lay less in refusing to believe, than in

neglecting to examine—an error so far from being fostered by, must have been in direct opposition to all his scientific habits.

Although I have been thus insisting upon the advantages of scientific study in a religious point of view, yet I must not be understood as holding the opinion that any of the mysteries of Revelation can receive a single ray of elucidation from the happiest discoveries of Science, or from the profoundest researches of Philosophy. I am afraid such an erroneous opinion is entertained by some, who seem to forget, what I would recommend you always to remember, that the exertion of human intellect is limited to the sphere of human experience; and that what Omnipotence has stooped to reveal, the unaided powers of the loftiest intellect could never have reached.

The most, indeed, that science and philosophy can do, is to dispossess the mind of those prejudices which foster error, and to enlarge its powers of attaining truth: and by presenting to it the most striking examples of irresistible evidence supporting incomprehensible truths, to lead us to repose on the conviction that the mysteriousness of a doctrine is no ground for its rejection, any more than the reasonableness of it is a sufficient test of its truth. The man of genuine science is not the man who opens the divine records, and picking out a passage here and another there, sets himself up to argue their divine authorship, because forsooth the doctrines conveyed by them are not such as *he thinks* are in accordance with what *he supposes* to be the enlightened legislature of Heaven: Philosophy has taught him that the data of all our reasoning are observed facts; and we have no access to the economy of the celestial world. In Revelation, as in science, our first business is to examine the evidence for its authenticity; and this being admitted, it would be in direct violation of all the canons of philosophy to reject it on account of any fancied internal discrepancies. Such fancied discrepancies have manifested themselves even in mathematical science, notwithstanding the refulgence of evidence with which its doctrines are surrounded; and although D'Alembert, a most profound mathematician, used to say that upon every subject it was permitted us to doubt except in mathematics, yet such, in his day, were the apparent inconsistencies in a certain branch of his favorite science, (the theory of Logarithms,) that he not only himself doubted, but held a public dispute with Leibnitz on the subject, and what is more, espoused the wrong side of the question? The apparent discrepancy was removed by Euler, who showed that a certain reconciling element had been overlooked, which ought to have been taken into consideration; and which, when introduced, adjusted the conflicting theories, and established harmony where discord seemed to prevail. Take

another example: The illustrious Lagrange, a man of the most stupendous mathematical powers, published to the world a certain analytical theorem, together with the reasoning by which it was supported. This reasoning was examined by contemporary and by succeeding mathematicians, and pronounced to be conclusive. Very recently, and about half a century after the publication of this theorem, another distinguished French analyst, Poinsot, arrived in the course of his investigations at a truth which appeared totally irreconcileable with the universally received formula of Lagrange. This unexpected circumstance led Poinsot to re-examine with great care the whole process of Lagrange, when he discovered that a very refined consideration had been lost sight of, which, when taken into account, rendered the two conclusions perfectly accordant. I know not how long Poinsot was in reconciling this seeming contradiction in a part of the "exact sciences;" but this I do know, that as he was a genuine man of science he did not withdraw his confidence in the truths of analysis because of this difficulty; for he felt assured that an adjusting element must exist somewhere, whether he could discover it or not; and his anxiety upon the subject must have been considerably diminished, from knowing that the discrepancy in question—even if never removed—was of that nature as to leave all the great truths of science undisturbed.

It would be easy to multiply instances such as these: conflicting doctrines still exist in some few parts of pure analysis: these are gradually becoming reconciled as science advances, that is as our views are becoming more and more enlarged, and our knowledge more and more accurate and comprehensive. Truth can never conflict with itself; and it can seem to do so only from our not knowing the whole truth,—"we know only in part."

I leave to you the obvious inference to be drawn from these examples. They prove to you that even in the most perfect of sciences, and one entirely within the grasp of human reason, apparent inconsistences sometimes present themselves, but which we know must arise either from some deficiency in our data, or else from the omission of some one of the various considerations which ought to accompany our reasonings, and control our deductions; and the recognition of which would restore all to harmony.

But while we maintain that mathematical studies thus operate on the mind as a corrective to scepticism, we must not be understood as implying that they favour any tendency to credulity. They are on the contrary equally opposed to both. The evidence required by Geometry, is precisely co-extensive with the proposition to be established; and however alluring and plausible a step further may be, we are imperatively restrained

within the bounds of the evidence. And hence mathematical knowledge has had a happy influence on the minds of men in their researches into the Phenomena of Nature; not merely by supplying them with efficient instruments of calculation, but also by restraining that disposition to be led by the ignis fatuus of fancy, beyond where the lamp of experience could guide, and to set up the filligree work of the imagination for the enduring designs of the Almighty. Most of the physical sciences, which are not completely under the dominion of the Mathematics, exhibit instances of this fault; although the great Bacon has rendered his name immortal by his successful efforts to purify science from such dross. I will give you an example of what I mean from the science of physical optics.

A celebrated French philosopher, now living, thus describes the phenomenon of vision. The rays of light, says he, enter the ball of the eye at the pupil, and crossing each other at a point beyond, reach a surface called the retina. On this surface a picture is produced which gives the forms, and even the colours, of external objects. *The impression produced on the retina is transmitted to the optic nerve, by which it is transmitted to the brain, the seat of the understanding*, and the sensation of vision follows.

Here now is a clear and succinct enumeration of circumstances: let us examine the evidence for their truth. The physiologists who have examined the organ of vision have ascertained, beyond a doubt, that there is such an impression made, or picture painted on the retina as is here described. They have also ascertained that the optic nerve, in a healthy state, and the brain, are both necessary to correct vision. They have proved, too, that when the optic nerve and the brain are in a sound state, when individuals are awake, and when the above-described impression is made on the retina, vision, or a sensation of sight, is produced; or we see the object the image of which is impressed on the retina. But there is nothing in the connexion between the picture on the retina, and our consciousness of perceiving the external object, to warrant the assertion that this impression is *transmitted* to the optic nerve, and subsequently *transmitted* to the brain. The picture on the retina is the ultimate physical circumstance which physiologists have yet traced when vision occurs; and between that and actual sensation and perception—or the mental act of seeing—all is darkness. We *know* that there are impressions on the retina; we *know* that in ordinary cases, when these are received, vision follows; but we *know not* how the bodily impression is connected with the mental perception; and to account for it by further impressions on the optic nerve, by impressions on the brain, and by this being the seat of intelligence, only serves to keep our ignorance out of view, and attempts to explain the incomprehensible connexion between the impression on the retina, and

the sensation of sight, by another equally incomprehensible connexion, viz. an impression on the brain with this sensation; of which connexion, however, we have no knowledge whatever.

There is indeed in this, as in every other physical inquiry, a barrier beyond which it is forbidden to human intellect to advance. There are in every direction some bounds to our knowledge, which, although they continually recede as investigation advances, can never be passed;—which equally exist, though their extent is different, for the most ignorant savage and the most enlightened philosopher; and at which every man, finding himself suddenly arrested, and being unable to explain the connexion between certain phenomena, feels the sentiment of wonder, and is compelled to reverence a Power the ways of which, he is thus made sensible, are inscrutable by a finite mind!* In investigating the phenomena of nature we may proceed from cause to cause to a considerable extent; but when we feel compelled to stop, let us humbly acknowledge the inadequacy of our own powers, and not presumptuously proclaim that we have arrived at the last link of the chain; for between this, and that where we feel obliged to terminate our progress, there may be an immeasurable interval—an interval which we are more

* See Dupin's 'Geometry applied to the Arts,' by Dr. Birkbeck, from whom the above remarks are taken.

likely to increase than to diminish, when we step beyond the bounds of experience, and speculate upon that which is hidden from our eyes.

Gentlemen, I here terminate these remarks. Enough has been said to show you that there are advantages connected with a course of scientific study, quite independent of all considerations about practical utility; considerations which, weighty as they are, I have not thought it necessary to bring before you on this occasion; because the advantages I have mentioned can be appreciated by you all, and are in themselves sufficient amply to reward your best exertions in the interesting employment which is now before you. It is scarcely necessary for me to say, that without such exertions on your part, none of these advantages can be fully secured; and while I indulge the confident expectation that my instructions here will receive from you that respectful attention which the dignity of the subject so eminently deserves, *you* also may rest assured, that the best efforts which I can command will be exerted to communicate that instruction in the simplest and most attractive form; and that the greatest satisfaction I can possibly receive, out of my connexion for a short period with you, will arise from the evidence which you shall give me, by your regularity and progress, that the aid which I am proud to offer you in the path of science is neither disregarded nor unfelt.

ADDRESS,

DELIVERED TO THE STUDENTS AT THE CLOSE OF THE COLLEGE SESSION 1838.

Gentlemen,

Nearly six months ago I had the pleasure of addressing you from this place. I have a lively recollection of the occasion; not merely because I see the same individuals again before me, but because of the permanent impression produced upon my own mind by the very marked and respectful attention with which you listened to the few observations I was then privileged to offer to you.

I advert to this circumstance now, in order to show you that I was not unmindful of it at the time, nor have yet forgotten it; and I advert to it thus publicly, that you may know how far exceeding all other gratifications to a teacher, is that which he receives from the quiet and earnest attention of his auditors; and that you may now have the satisfaction of feeling,—if during the bygone session you have in your several class-rooms given this earnest attention to your professors,—that you have thus afforded to them a testimony of your respect for their persons, and of your appreciation of their labours, far more acceptable,—far more encouraging, than any other you can possibly bestow.

I have just stated, that I have the pleasure of again meeting, at the close of the session, the same individuals whom I addressed at its commencement. This statement, thanks to a protecting Providence, requires but little qualification. I believe not one student who occupied a seat upon those benches at the beginning of the session, has been called from the work upon which he then entered,—that of preparing for time, to realise eternity. This is matter of devout thankfulness to us all,—that during a season of such unusual severity we have been spared, and are permitted now to meet, without having to lament a like occurrence to that which last year deprived the Institution of one of its most distinguished students,—you, of one of the most beloved of your companions,—and the Christian Church of one of the most promising of her sons.

But, although I am thus speaking to the same persons as on the former occasion, I feel that I now address you under very dissimilar circumstances. You appear before me in a different character,—you occupy a different position,—you present a different aspect. *You*,

as well as I, *feel* this. We met before, at the commencement of a new stage in your intellectual career; that stage has been passed over; and we now assemble, at its termination. I regarded the majority of you *then*, in reference to the engagements upon which you were about to enter, as standing in one common predicament, as occupying the same level. It was not in the power of any one here to have arranged you upon those benches according to any scale of intellectual precedency. You could not, if you had tried, have fallen into that arrangement yourselves. No one of you could have estimated accurately his own position in such a scale relatively to the individual beside him, in any one of the classes into which you were simultaneously to enter. You all entered under common conditions,—in most cases with like capabilities,—and no doubt with like resolutions and like hopes. During the period which has intervened you have all enjoyed the same advantages,—have been admitted to the same privileges,—have had access to the same sources of instruction. Allow me to ask you, has the operation of these like causes produced like effects? and, as at the *opening* of the session I addressed indiscriminately to all the language of advice and encouragement, am I warranted, at the *close* of the session, in awarding, indiscriminately to all, the well-merited tribute of praise? Think you that the arrangement to which I have just adverted would be as impracticable *now* as it would have been when we last assembled? It would surely be no difficult matter now for each Professor to subdivide his class into groups, and to assign to each group its appropriate place in such an arrangement. You yourselves might now construct the scale, and could decide with tolerable accuracy who should be advanced to the top, and who should descend to zero.

Like then as you are to the persons whom I addressed before, I cannot conceal from myself, nor can you be unconscious, that I am in reality *now* addressing two distinct classes of students;—the one class consisting of those who have justified their title to the appellation which they bear; the other, and I would fain hope and believe, a very small class, of those who to the name have neglected to attach the character and habits which that name implies.

If there be any of this description amongst you,—and such is the perversity of human nature, that in so large a body of students I am justified in supposing there may be some,—some who are the victims of remorse, when they look back on time misspent,—opportunities disregarded,—resolutions unperformed; or of self-reproach and apprehension, when they look forward to expectant friends and anxious parents: I say, if there be one so circumstanced present, let me urge upon him to adopt now, and to act upon, this resolution;—let him

resolve to lose no more time in lamenting over that which he has lost already; but, instead of this, let him, when he reaches his retirement, instantly and earnestly betake himself to his neglected studies. By a judicious appropriation of the time now at his command, he may atone for the error he has committed. Let him but summon energy enough to begin,—he will soon acquire ardour enough to proceed.

But to the opposite class of students,—those who have earned for themselves the distinguished notice which they have just received,—to *you*, who have so honorably sustained the character you assumed when we last met,—and who, despising alike the allurements of indolence and the temptations of pleasure, have perseveringly laboured to the end,—to you, I merely say, that in the placid satisfaction you now enjoy, and which ever attends successful intellectual effort,—you feel, while I thus address you, a more exalted pleasure than any eulogium of mine can possibly convey. You will this day leave the scene of your exertions with enviable emotions; knowing that you have been *successful* because you have been *laborious*,—in other words, because you have deserved success; and justly attributing the distinction which has been conferred upon you to your own persevering efforts. Some of you, to whom I have had the pleasure of awarding premiums to-day, have not obtained this honour without much extra labour;

nor could you have obtained it, without devoting to your scientific inquiries a considerable portion of that time usually allotted to repose. For such exertion the reward you have just received is, I know, altogether inadequate. But if in the course of these solitary researches, you have happened upon a single truth unknown to you before, you have had your reward in the mental approbation that has followed; for remember, the discovery of Truth is the highest achievement to which a mortal can aspire,—the approbation of his own mind, the highest gratification a mortal can enjoy.

Having thus tasted the pleasure, and received some of the distinctions of intellectual acquirements, it is but reasonable to expect that you will thereby be stimulated to further application. The season of cessation from collegiate duties which has now arrived, must not be regarded even by you who have signalised yourselves in your different classes, as a season of mental inactivity. After a moderate repose, you will, I trust, return refreshed to your several pursuits. You will find yourselves profitably employed in revising the acquisitions you have made, and in carefully examining the stores you have accumulated.

In addition to this, you must aim at making new accessions. If while drinking at the sources of instruction here, you have not at the same time imbibed the spirit of self-cultivation, sufficient to excite your own indepen-

dent exertions when those sources are withdrawn, they will have been opened to you to but little purpose; and there will be reason to fear that we have mistaken a burdened memory for a fertilized and improved mind. The great object of education is to originate an earnest desire after knowledge, and to foster the habit of private and solitary study. Without such a habit, intellectual eminence can never be attained. It is in this that the true secret of what is called *genius* consists, —a name that only serves to conceal from us the continuous effort,—the untiring perseverance, and the days and nights of solitary labour, to which the attainment of excellence is always due.

To *continue* distinguished, therefore, you must add to your college acquirements the results of your own independent researches, the product of your own individual efforts; and in urging this upon you I am reminded, that, with very few exceptions, you are each destined for the important duties of a Christian minister. You may be placed in circumstances in which you will be called upon to prove the authority of the commission which you hold, or to defend the doctrines which that commission embodies; to do this in a proper spirit, and with allowable weapons, it is requisite that you submit your minds to philosophic training, in order that you may acquire a readiness in distinguishing scientific argument from scientific sophistry; in order, moreover, that your knowledge of the apparent discrepancies, the real difficulties, and the real mysteries of science, may subserve the purposes of your higher vocation.

I have spoken of the *mysteries* of science; and if I be justified in calling that mysterious which is confessedly incomprehensible, I use the term advisedly. Nobody, for instance, can comprehend *infinity*, yet this is an ordinary element in some of the higher inquiries of science. Those of you who have attended the Senior Mathematical Course know this. You know, also, that some of the most striking and valuable truths investigated in that course, are involved in, and are educed from expressions altogether incomprehensible, and called accordingly *imaginary* or *impossible* quantities; and I may remark, that some of the sublimest results of Physical Astronomy have been reached through the aid of the same class of incomprehensible formulas. Of late years, the discoveries of science have had, or are at least fitted to have, a powerful effect in nullifying some of the favorite philosophical arguments of Infidels, and in removing objections to Scripture, on the score of alleged inaccuracy in its historical and physical details.

You have all heard of the Calculating Machine. It is an engine contrived by a very eminent philosopher, for the purpose of enabling us to submit long and intricate calculations to a series of wheels and pinions. There are some facts connected with that remarkable

piece of mechanism which deserve special notice, and which may perhaps not inaptly be adverted to here, in justification of the statement already made respecting the bearings of modern science upon infidel arguments.

Conceive yourselves to be seated before the Calculating engine, and to be observing the numbers successively presented to you upon its face. These numbers you find to follow each other at fixed intervals, and in strict obedience to a certain law, unknown to you the observers, but which the contriver has chosen previously to impress upon its movements. You may, if you please, fancy the necessary adjustments to have been made in your presence, and the engine to have been set going before you; or you may imagine your attention to have been directed to it *after* its operations have commenced; but you are to understand that, the adjustment once made, the machine is then left entirely to itself, and cannot act but in conformity to the law originally impressed. Suppose this law to be such as to cause the series of odd numbers, 1, 3, 5, 7, &c. to pass before you in regular successive order,—admit that you have steadily watched these uninterrupted appearances till 1000 terms have passed before you; will you feel any doubt as to the numerical law whose indications you have observed in so many individual instances? Could you not predict, with considerable confidence, what the 10,000th term would be,—that it would be the 10,000th number in the series 1, 3, 5, 7, &c.? Imagine such prediction ventured,—you wait for its fulfilment, and when the 10,000th term arrives, the odd number you had foretold actually presents itself upon the face of the machine. This verification of your prediction excites no wonder; you had already observed the phenomena of the engine with sufficient care and constancy, to enable you to infer, after so lengthened an experience, the numerical aw under which every number presented by it must come, and that so long as the original adjustments remain unaltered and unimpaired, an odd number must make its appearance, and take its proper place in the series;—the only object of your wonder would be the intellect of the contriver.

But what would be your astonishment to find, after this undeviating regularity had been maintained, not merely through a 1000, or 10,000 terms, but through many millions of terms, that an *even number* presented itself! and that then the *odd* numbers were immediately resumed, and the series indefinitely continued? What would you say of such an unexpected occurrence, of an event so opposed to all your past experience, and to the experience of every one who had watched the machine through such an immense period? Or what, in consistency, ought David Hume to say of a phenomenon so anomalous, so irreconcileable with the *law* of the machine? Here is the violation of a law which has operated

faithfully and undeviatingly throughout all experience,—an experience which is immeasurably against the probability of such a violation. In such circumstances, Hume, with his notions of the weight of experience, and of the immutability of the laws of phenomena, would find himself in great perplexity. But in *this* case, the contriver of the machine might remove his perplexity at once, by stepping forward and telling him, that wide as his induction of particulars had been, he had nevertheless generalized too hastily, and that his *hypothetical law* was *not* the law that had been impressed upon the machine. The true law having been such as *necessarily* to involve that particular case which he had regarded as a violation! And such would be the *real* explanation of the apparent anomaly. Mr. Babbage can so adjust his engine, that, when left entirely to the government of those mechanical laws which are brought into action, the very phenomena I have described shall actually take place! I regard this, gentlemen, as a very remarkable, and a very valuable fact,—one altogether uncontemplated, originally, by the contriver himself, but which he has turned to admirable account in reference to Hume's celebrated argument upon Miracles.*

Equally valuable and unexpected have been the results of recent inquiries into the mechanical laws of light.

* See a Note on Hume's argument at the end.

Who would have expected, that researches in physical optics could have completely overturned the most formidable objection that had ever been urged against the Mosaic account of the creation! I refer, of course, to the revealed fact of the creation of light before that of the luminous body. The researches to which I allude, and which for the last twenty years have been prosecuted, both theoretically and experimentally, by the greatest philosophers in Europe, have led to a vast variety of results, which all unite in proving that light consists, not in emanations from the luminous body, as Newton supposed, but in the vibrations of a fluid which pervades all nature; and that the exciting of these vibrations is the only office performed by the luminary. The creation of light, therefore, before the creation of the sun, was but the introduction of the machinery before that of the agent which was to put it into motion. Had the luminous body been created first, there would have been a splendid example of premature and wasted power, for its light-exciting energies must have expended themselves in vain.*

Such are some of the collateral advantages attendant upon scientific pursuits. Many more might be easily adduced, but I will not fatigue you with a further enumeration. Enough has been said to show you, that the more extensively you range over the fertile field of

* See note (C) at the end.

science,—provided only that truth be the object of your search,—the more likely are you to advance in religion and morality. Enough has been said to show you that science is no friend to that presumption, with which ignorance dogmatically pronounces upon that which is above the reach of human comprehension; and, therefore, enough has been said to show you how it comes to pass, that those most conspicuous for their intellectual attainments, have always been remarkable for their modesty and humility. Newton and Laplace were eminent examples of the union of profound science with modest pretensions. The Christian humility of the former is well known; and the last words which his great successor uttered were these, "The amount of our knowledge, how little it is,—the extent of our ignorance, how immense!" But I will not detain you with further remarks.—You are, no doubt, anxious to leave the scene of your labours, and to taste that repose which many of you so much require, and so well deserve. Give me your earnest attention, while I address to you one parting word of exhortation and of hope. Receive it not as the mere formal conclusion of a ceremony which custom has rendered necessary; but receive it as the language of sincerity and truth, dictated by a feeling of concern for your own prosperity, and for the reputation of the Seminary with which I have the honour to be connected.

Gentlemen,—We feel—I speak for my colleagues as well as for myself,—we feel an interest in your welfare that is not circumscribed by the walls of this Institution,—an interest which will now accompany you to your homes, and which will hereafter follow you among the duties and engagements of active life. We do not forget, that whatever reputation we, as a body, can hope to enjoy,—or that our plan of education, as a system, can acquire,—must flow through YOU. In justice to us, we expect you to give evidence that we labour faithfully in our vocation. In justice to yourselves and to your country, we exhort you to show that that labour is expended upon no ungenial soil. Some of you we shall in a few months meet again, as candidates for the *final*, and what should be regarded as the most *honorable*, distinction that this Institution can confer. We shall then have to bid you farewell, in your capacity of students: we shall do so with a sincere and deep anxiety, as well for your prosperity, as for our own credit. But that anxiety will be amply repaid, if in after years we be privileged to see but *one* of you who have been distinguished to-day, justify, by your character and well-earned eminence, those high expectations which, upon occasions like these, *we* cannot help indulging. When distributing our prizes in this Hall, *we* would fain regard them, not as rewards for effort expended, but as stimuli to labours yet to be begun; not as honours

already secured, but as types of distinction yet to be attained. We hope to hear of you in the walks of literature and science again: we hope to see you combating with difficulties of a higher order,—displaying your intellectual energies on a wider theatre,—receiving distinctions upon a larger scale. We hope to see you emerge from the multitude around you,—rise above the level of mediocrity, and finally attain to that elevated position that will place you upon a footing with those whose names have shed a lustre over the page of their country's history, and whose virtues and talents have adorned and dignified the human character.

NOTE (A.)

NECESSARY TRUTHS.

THE statement that the truths of geometry are *necessary* truths, has often been denied by metaphysicians, who maintain that the foundations of geometry are purely hypothetical, and have no real existence, either in external nature or even in the mind itself. Dugald Stewart was, I believe, the first philosopher who unequivocally affirmed and attempted to establish this doctrine; and in a recent publication of great learning and ability, the same view is insisted upon with still greater emphasis. But I consider it to be altogether erroneous, however consistent it may be with the supposition that all our knowledge is furnished by observation and experience. The writer just referred to says—" The points, lines, circles, and squares, which any one has in his mind, are, (I apprehend,) simply copies of the points, lines, circles, and squares, which he has known in his experience;" and again—" neither in nature nor in the human mind, do there exist any objects exactly corresponding to the definitions of geometry." * Now this appears to me to be quite contrary to fact, and to impose arbitrary limitations upon our powers of conception, which experience proves to have no real existence. I admit that observation and experiment are the foundation of all our knowledge; but I by no means admit that our knowledge consists of no more than this; for I recognize a superstructure as well as a foun-

* Mill's System of Logic, vol. i, pp. 297-98.

dation. If a rude and imperfect sphere be presented to me, and I observe that by the successive efforts of art these imperfections become more and more diminished, I can mentally anticipate the desired consummation, and *conceive* the perfect sphere; though art should never be able practically to arrive at it. The same may be said in reference to a circle, a square, &c., and I can thus acquire the geometrical conception from only the approximation to the reality which art presents. Can the mind conceive, in such cases, no more than the hand can execute? Further: it is the ultimate aim of art to realize the geometrical conception, or to elaborate its productions till they fulfil the geometrical conditions: and may it not be asked—when this perfection is so nearly reached that the senses are unable to appreciate the supposed defect—May it not be asked—Whence do you learn that such defect really exists? the senses cannot discover it;—what can? Will any one undertake to point out the defect from sphericity of a well-turned billiard ball? if not, on what ground does he pronounce it imperfect? does he not, in so doing, go beyond experience? We cannot, I think, upon strictly philosophical grounds, deny geometrical forms, even to external existences, much less can we deny the conceptions of such forms in the mind. And it seems scarcely a figure of speech to say that the perfect sphere existed in the solid material, whether the maker of the billiard-ball adverted to, succeed in bringing it forth or not.

The axioms and other truths of geometry are thus *necessary* truths:—not necessary merely as logical deductions from arbitrary hypotheses, but necessary in reference to actual existences—existences which have place at least in the mind if not in external nature.

NOTE (B.)

ON HUME'S ARGUMENT AGAINST THE POSSIBILITY OF MIRACLES.

THE celebrated argument of Hume against the performance of miracles has been combated by many persons. There is scarcely a writer on the "Evidences" who has not attempted its refutation; and, of late years, a good deal of masterly reasoning has been brought to bear on this interesting topic by Dr. Chalmers, Lord Brougham, and Mr. Babbage—the last of whom, as far as I know, was the first to give to these reasonings a strictly mathematical form, and to express their results in definite numerical measures.

The position which Hume labours to establish is in substance this:—That no amount of human testimony in favour of the performance of a miracle, can ever render its occurrence so highly *probable*, as the uniform experience of mankind, against a violation of "the laws of nature," renders it *improbable*. And the object of preceding writers has always been to show, in contravention of this doctrine, that a comparative limited number of witnesses of ordinary veracity, and without collusion, bearing concurrent testimony to the event, are sufficient to render the probability in favour of its occurrence, far greater than the *à priori* probability against it, as inferred from the uniform experience of all mankind. It is the principal object of these observations, by presenting the matter in a light somewhat different from that in which it has hitherto been viewed, to show that the above-mentioned condition, as to the veracity of the witnesses, may be altogether dispensed with; and that if these witnesses, without collusion, come voluntarily forward to affirm the occurrence of a miracle, and all testify to the *same* miracle, the simple fact of this concurrence in their testimony—if they be at all numerous—will give a probability to the truth of their statement which is altogether irresistible, *however abandoned the character of the witnesses may be.*

It seems to be the more desirable to do this, since the final impression intended to be made by the so-called argument of Hume is, that the witnesses for the Gospel miracles were either impostors or deluded enthusiasts; and an essential element in the opposing hypothesis, hitherto adopted, is, that they were at least in some degree honest and trustworthy. Let us then concede that the witnesses were altogether unworthy of the slightest confidence—that one comes forward, bent upon deception, and affirms that he is eye-witness to the performance of a miracle—the raising of a dead man to life, for instance, and let ten other persons, with the same disposition to deceive, but without collusion, testify to the same thing. Now let us assume that these ten persons were limited within the very narrow range of only ten fabrications suitable to their purposes of fraud, the probability that they would all fix upon the particular miracle mentioned is the tenth power of 1-10; that is, it is $\frac{1}{10000000000}$. If, instead of *ten* other persons, there were *twelve*, the probability would be $\frac{1}{1000000000000}$; that is, the *odds* against the occurrence of this supposed uniformity of testimony is within a unit of a *million millions* to *one*. Now, supposing, according to Mr. Babbage and Laplace, that the origin of the

human race was about 6000 years ago, and that 30 years is the average duration of a generation, 200 generations must have passed away; and allowing that the average population of the earth has been a thousand millions, we find that there have lived and died, since the creation, about two hundred thousand millions of individuals. The experience of all these, in favour of the non-occurrence of a miracle, is therefore *two hundred thousand millions to one.* We have seen above, that if only thirteen individuals bear independent testimony to the fact—even supposing that they have only nine other events to choose from—the probability that that fact is *not* a fabrication is a *million millions to one*—a probability far surpassing the former; and it will be observed that this result is wholly independent of the character of the witnesses either as to honesty or intellect.

The foregoing conclusion is incontrovertible; and like all the other deductions from the theory of probabilities, will be found in strict accordance with the suggestions of common sense. If a notorious liar meet us in the street, and inform us that a murder has just been committed at a certain place, we give little or no credence to his statement; but if a second person, equally void of integrity, and who, we know, cannot have communicated with the former, tell us the same story, we cannot resist a strong impression in favour of its truth—we place no value upon the veracity of our informants, but a very high one upon the improbability of two fabricators of falsehoods independently inventing the same thing at the same time; if a third person, of like character, repeat to us the information, we place as much reliance on the truth of the statement as if it had been made by a person of unquestionable veracity. The evidence of bad characters is frequently rejected in courts of justice, on the ground that they are not to be believed on oath; yet, if several of these bear independent testimony to the same event, out of a number of other events equally likely to suggest themselves, or to have occurred, we see that a high degree of probability would attach to the statement.

The other method of refuting Hume will be found very fully and ably discussed in Dr. Chalmers's volume on the Evidences of Christianity, and in Mr. Babbage's Ninth Bridgewater Treatise. In this latter work, the same numerical precision is given to the reasoning as is attempted in the remarks above; but, from a little oversight in the details of the investigation, a considerable error occurs in the final result,—the necessary number of witnesses appearing to be twenty-five, whereas, just as in the other view of the question, the correct number is thirteen.

We have thus two independent methods of refuting the fallacy of Hume,—that no amount of human testimony *for*, can counterbalance the weight attached to the uniform experience of mankind *against*, a "departure from the course of nature."

The first argument shows that if thirteen impostors, or thirteen insane or deluded persons affirm, without collusion, that they were eye-witnesses of a miracle, supposing even that only ten pretended miracles could be invented, or suggested to their minds, the probability that they affirm the truth is *five times* as great as the probability for the constancy of the "laws of nature," as deduced from all past experience.

The second argument shows, that if thirteen witnesses, whose veracity is such that they each tell *one* falsehood in every *ten* statements they utter, testify, without collusion, to the occurrence of a *specified* miracle, the probability of the

truth of their statement is, as before, five times the probability for the constancy of nature.*

In this latter argument the condition of non-collusion, which is always introduced into the premises, might be suppressed; and thus the force of the conclusion augmented; for the same amount of honesty that would prevent two or more individuals from fabricating a falsehood independently, would prevent their doing so in concert; but if this be disputed, it will at least be conceded, that the supposed veracity of the witnesses must render collusion highly improbable, a fact which proportionately narrows the demand of the postulate.

It is evidently the drift of Hume's argument to establish the position that the occurrence of a miraculous event ought to be believed only by those who were the actual eye-witnesses of it. Now, admitting it to be necessary, although certainly not sufficient, that the bearer of a divine message should authenticate his mission by the working of a miracle, it would follow, in order that all might receive this message, that all should be eye-witnesses of the miracle; and, moreover, as traditional evidence, and the experience of others, whether verbal or recorded, goes for nothing, it is further necessary that the miracle be repeated to each succeeding generation, and so on to the end of time. But a phenomenon that thus presents itself to the observation of mankind, age after age, with unfailing certainty, is not a *miracle*, but *a law of Nature;* if not, what is it that distinguishes the *law* from the *violation?*

The partisans of Hume might think to escape this dilemma

* The mathematical details, whence this latter conclusion is deduced are not suited to this place; they will be found in the Appendix to the fourth edition of the Author's 'Treatise on Algebra.'

by proposing that *the same miracle* should not be repeated age after age, but that there should be a succession of *different* miracles, or of violations of law in different departments of nature. But these so-called violations of law would be destructive of all law; for no department of nature being uniform in its phenomena, no laws could be recognized, and consequently no miracle.

It is absolutely essential, therefore, in order that miracles may retain their character as such, and produce their intended effect upon our belief, that they be of infrequent occurrence, and be handed down from age to age as the recorded testimony of eye-witnesses. And it is strictly true, that the force of human testimony, in a matter of this kind, comes at length to exceed that of actual and personal experience; since, as is just shown, miracles, which every successive generation is called upon to witness, either at length fall in with the laws of nature, and thus lose their character and effect; or else they render those laws so obscure and ambiguous, as to leave the breach of them undistinguishable from the performance.

The usual definition of a miracle, that it is "a *violation* of a law of nature," is, I think, not sufficiently comprehensive. There may be an obvious and direct putting forth of divine power, in a manner altogether unprecedented, without the violation of any existing law: it is sufficient that the law be made to exhibit itself with extraordinary intensity, or under circumstances never before experienced, in order to produce effects to which past observation furnishes no parallel. Thus the stationary position, or the retrograde motion, of the sun—adverted to at page 16—does not necessarily imply the violation of any law. And even the deluge itself may have been the result of laws still in operation, but acting in circumstances altogether unique. Who can prove that the proximate cause

of the catastrophe was not the collision of a comet? Our earth is, in fact, threatened with a repetition of the calamity from such a cause, although certainly at a very remote period: the harbinger of its destruction is the comet of Encke. Concerning its approach to our own planet, Olbers has computed that, in the course of 88,000 years, this comet will come as near to us as the moon. That in four millions of years it will pass at the distance of about 7700 geographical miles, when, if its attraction should equal that of the earth, the waters of the ocean will be elevated 13,000 feet, that is, above all the European mountains except Mont Blanc. The inhabitants of the Andes and the Himalaya mountains, therefore, would alone be able to escape such a deluge, which would probably leave upon our globe records of its occurrence, similar to those discoverable at the present day. After a lapse of 219 millions of years, according to the calculations of the same astronomer, an actual collision will take place between this comet and the earth, severe enough to shatter its external crust, alter the elements of its orbit, and annihilate the various species of animated beings dwelling on its surface.* Whatever be the value of these conclusions, they at least show that the supposition of the deluge being caused by a comet is by no means chimerical; though our ignorance of the actual constitution of comets should preclude our pronouncing with certainty as to the extent of devastation which a collision would cause.

Dr. Halley computed the period of the remarkable comet of 1680 to be about 575 years, and thence inferred that it must have appeared near the time of the deluge, and that it probably occasioned the catastrophe by the contact of its tail,—an appendage which, in 1680, exceeded in length the distance of the

* See Milne's Essay on Comets, 1828, p. 131.

sun from the earth, it having been computed to reach upwards of a hundred millions of miles!

It may not be improper to observe in conclusion, in reference to the arguments in the former part of this note, that the doctrine of probabilities, on which they are based, is a doctrine of the most unerring accuracy; and is not a theory of doubt and uncertainty, as the general reader might, from its name, suppose. It is in strict conformity to the rules it suggests, that the business of all life-assurance and annuity offices is conducted; and the failure of an office so conducted would be a phenomenon in the commercial world, provided, that is, that the office really got custom. So true is this, that if a set of adventurers were to start an office of this kind, and, by offering superior advantages to the public, were to depart from the strict results of the theory of probabilities, its ruin might be predicted with the utmost confidence. A few years ago such an office was opened in London, under the title of the "Independent West Middlesex;" the able and ingenious writer of the work on Annuities, recently published by the Society for Diffusing Useful Knowledge, predicted its downfall; and its ruin was actually accomplished while his remarks were passing through the press.

NOTE (C.)

ON THE LUMINIFEROUS ETHER AND RESISTING MEDIUM.

The comet of Encke, referred to in the preceding note, was first seen by Mechain and Messier, in 1786; but as these observers took only two observations of its position, the elements of its orbit could not be determined. It was rediscovered by Miss Herschel in 1795, and was also observed by several European astronomers. In 1805 it was again discovered by three astronomers simultaneously—Pons, Huth, and Bouvard; but no suspicion was entertained that this was the comet of the former years. In 1819 Pons a second time detected it, regarding it as a *fourth* new comet; but Encke discovered the identity of the four, and was the first to calculate its period and to predict its future appearances. These predictions, however, though every known cause of disturbance has been cautiously examined and estimated, have been invariably anticipated:—the comet always arriving at its perihelion about two days sooner than the time of its predicted return. This has led Encke to the conclusion that the motion of the comet is not in a vacuum, as is assumed in the computation, but that it has to contend with a *resisting medium*, which, by hindering its onward progress, gives the sun more command over it. It is thus drawn nearer to the centre of attraction, the orbit is consequently contracted, and thus the return to the perihelion expedited. Upon the hypothesis of a medium whose density varies inversely as the square of the distance from the sun, Encke finds that the acceleration of the comet will be fully accounted for; and the prevailing opinion among astronomers is, that such a medium must needs exist. It is remarkable, however, that the comet of Halley gives not the slightest indication of the existence of such a medium, although one might expect that its accumulated effect during seventy-six years would have told very perceptibly upon its period.

The undulatory theory of light—to which reference is made in the text—has tended to confirm Encke's views; for it appears to be thought essential to that theory that the luminiferous ether should possess *all* the properties of a fluid, and offer resistance. But if the phenomena of light can be accounted for, without any reference to the inquiry whether what we call the luminiferous ether offers resistance to bodies passing through it or not, I must confess that the assumption of such resistance seems to me to be gratuitous. For the purpose of explaining phenomena, we speak of the electric fluid, the magnetic fluid, &c., without ever including the idea of weight or resistance. In fact, we know nothing whatever of the agents which we thus name, beyond the phenomena actually exhibited. The comet of Encke, like all other periodical comets, is observed to undergo considerable physical changes during its revolution. It is in itself a comparatively small nebulous body, destitute of the usual cometary appendage—a tail; but its dimensions rapidly contract as it approaches its perihelion, and then as rapidly dilate upon its departure into more distant regions. Encke, and many other astronomers, regard this circumstance as an additional indication of a resisting medium, subjecting the comet to a greater depression as it comes into the denser strata in the vicinity of the sun; but the celebrated Bessel considers that the change of volume of the comet may as well be attributed to a real loss of the constituent molecules of such an evaporable body, as to any condensation it would undergo in its passage through the regions of ether. Sir John Herschel offers a suggestion somewhat analogous to this:—"It is very possible," says he, "that the change may consist in no real expansion or condensation of volume (further than is due to the convergence or divergence of the different parabolas described by each of its molecules to or from a common vertex), but may rather indicate the alternate conversion of evaporable materials in the upper regions of a transparent atmosphere into the states of visible cloud and invisible gas, by the mere effects of heat and cold.

I must confess that, to me, these latter (changes of temperature) appear to be the principal, if not the only, causes of the observed physical mutations which all comets undergo. The alterations themselves are matters of observation, and are universally admitted. Can it be reasonably imagined that such continual changes of figure and volume should be going on without causing a displacement of the centre of gravity of the mass? If the centre of gravity be not assumed to remain undisturbed amidst all these changes of form and bulk, there will probably be no necessity to call in the aid of a resisting medium to account for the discrepancy between observation and theory, as to the period of a comet's revolution.*

It may be proper to add, that the remark in the text—as to the creation of a luminiferous ether before that of the sun—is

* See the Author's edition of Bonnycastle's Astronomy, p. 248.

not offered as a positive statement of the divine procedure—that it was this ether that the command "Let there be light," actually summoned into existence; but only as an answer to a philosophical objection: the hypothesis of such an ether being now pretty generally admitted into science, for the purpose of connecting and explaining the various phenomena of light. It may be very difficult, or even impossible to prove, beyond all question, that the ether really exists in nature. But, seeing the number of phenomena for which the hypothesis of such existence satisfactorily accounts, it would be still more difficult to prove the non-existence of it: yet this must be done before the philosophical objection adverted to can have any weight.

THE END.

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